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Measuring Zak phase in room-temperature atoms

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Abstract

Cold atoms provide a flexible platform for synthesizing and characterizing topological matter, where geometric phases play a central role. However, cold atoms are intrinsically prone to thermal noise, which can overwhelm the topological response and hamper promised applications. On the other hand, geometric phases also determine the energy spectra of particles subjected to a static force, based on the polarization relation between Wannier-Stark ladders and geometric Zak phases. By exploiting this relation, we develop a method to extract geometric phases from energy spectra of room-temperature superradiance lattices, which are momentum-space lattices of timed Dicke states. In such momentum-space lattices the thermal motion of atoms, instead of being a source of noise, provides effective forces which lead to spectroscopic signatures of the Zak phases. We measure Zak phases directly from the anti-crossings between Wannier-Stark ladders in the Doppler-broadened absorption spectra of superradiance lattices. Our approach paves the way of measuring topological invariants and developing their applications in room-temperature atoms.

Introduction

Topological matter has promising applications in noise resilient devices and quantum information processing^{1,2}, thanks to the robust topological response guaranteed by global geometric quantities of the Bloch bands, namely, the topological invariants^{3–6}. These invariants change stepwisely only when the bulk goes through a topological phase transition, which involves band gap closing and reopening. Characterizing topological invariants is a central task in synthesizing and simulating topological phases of matter. They are usually measured by the response from gapless edge states based on the bulk-edge correspondence. However, edges are not always available in atomic quantum simulators⁷. On the other hand, the topological invariants are proportional to the geometric

phases accumulated across a whole Brillouin zone. We can also measure the geometric phases from the bulk energy bands to obtain the topological invariants. Along this line techniques based on reciprocal-space interference⁸, quench dynamics^{9–12}, and Hall transport^{13,14} have been developed in atomic simulators.

Previous experiments of determining geometric phases from bulk response in ultracold atoms rely on dynamic evolution or adiabatic manipulation^{9–16}. It has been shown that geometric phases can be obtained from the energy spectra of electrons in a constant force^{15–18}, which turns the Bloch energy bands into Wannier-Stark ladders (WSLs) with equidistant discrete energies. In one-dimensional (1D) systems, the displacement of the energies of the WSLs is proportional to the applied force and the positions of the Wannier centers (WCs)¹⁹, which reflect the values of one-dimensional geometric phases, i.e., the Zak phases³ of the energy bands^{20–22}, as schematically illustrated in Fig. 1a, b. Here we develop and implement such a spectroscopic method to retrieve Zak phases. We show that this method enables the determination of Zak phases in room-temperature atoms, which greatly improves the accessibility of topological matters and facilitates their applications.

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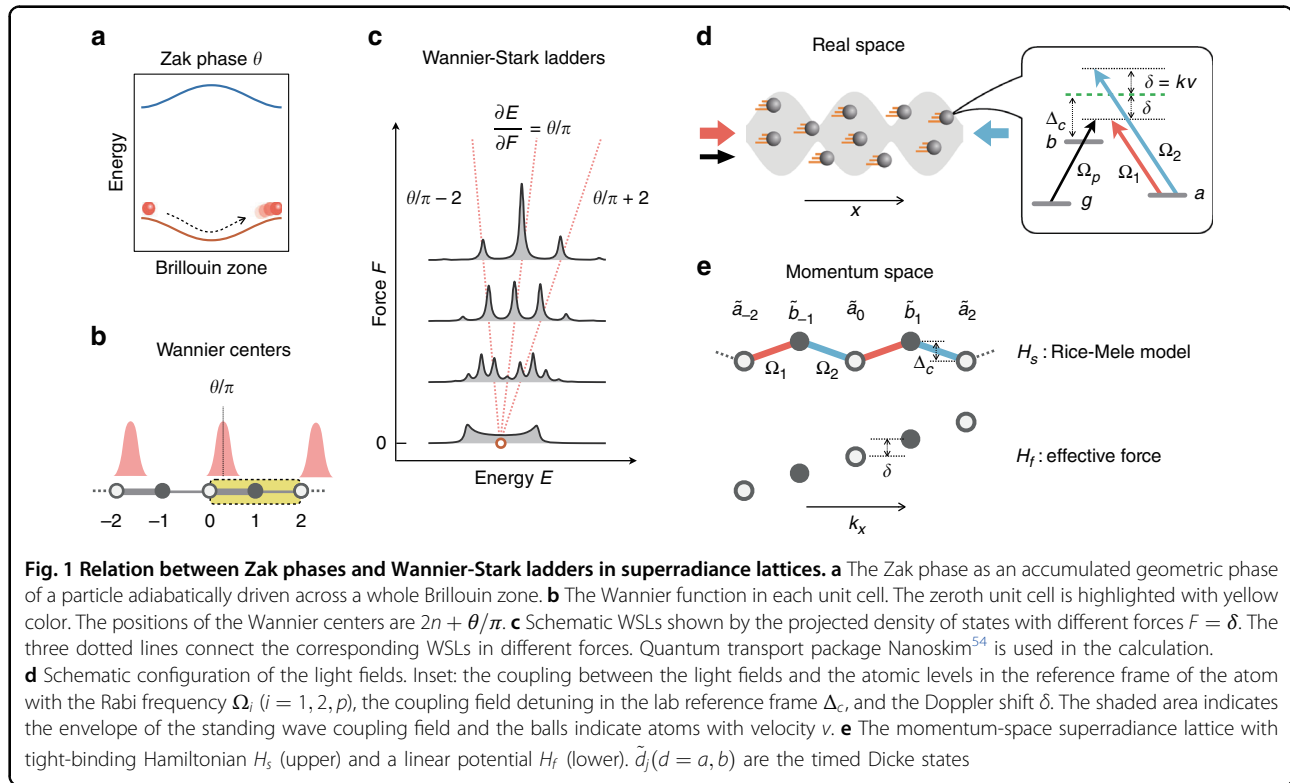
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We reconstruct the Zak phases of the Rice-Mele (RM) model through the anti-crossing between the WSLs in a 1D tight-binding lattice^{23–25} of timed Dicke states²⁶:

$$|b(k)\rangle = \frac{1}{\sqrt{N}} \sum_m e^{ik \cdot r_m} |g_1, g_2, \dots, b_m, \dots, g_N\rangle \quad (1)$$

i.e., single-photon collective excitations of an ensemble of N atoms (here r_m is the position of the m th atom with the ground state $|g_m\rangle$ and an excited state $|b_m\rangle$), which can be created by coherent laser fields that transfer momentum $\hbar k$ to the atoms. We introduce multiple laser fields to couple $|b\rangle$ to another atomic state $|a\rangle$ such that timed Dicke states with discrete k values form a momentum-space tight-binding lattice, which is coined the superradiance lattice. When the momentum of a timed Dicke state matches that of light, directional superradiant emission of radiation can be observed²⁷, which provides a convenient way to measure the lattice transport. A substantial difference between the momentum-space superradiance lattice and conventional real-space lattices in solids is that the Brillouin zone (BZ) of the superradiance lattice is in real space, where atoms at different positions can be independently diagonalized by only considering the local field strengths of a spatially periodic coupling field (here a standing wave). The positions of atoms play the same role of the lattice momenta of electrons in solids. A remarkable consequence is that atoms in motion travel through the real-space BZs

periodically, following the same dynamics of electrons subjected to a constant electric field. Therefore, atomic motion provides an effective electric field (or more precisely a constant force) for the excitations in superradiance lattices.

The Zak phases of a tight-binding lattice can be obtained from the WSLs when an effective force is introduced in the lattice. Since the motion of atoms provides such an effective force in superradiance lattices, we can take advantage of the thermal motion of the atoms to read out the Zak phases from the energy spectra. In particular, different velocity groups of thermal atoms provide a set of continuous values of the effective force, which results in a set of WSLs with displaced energies proportional the Zak phases^{15,16} (see Fig. 1c). A key to extract the Zak phases from the absorption spectra of superradiance lattices is that WSLs from the two energy bands have anti-crossings when they approach the same energy, which results in absorption peaks and dips. The Zak phases can be obtained from simple geometric relations between the locations of the anti-crossing points and the band centers. We investigate in detail two celebrated versions of the RM model, the Semenoff insulator and the Su-Schrieffer-Heeger (SSH) model^{28,29}. We also demonstrate the Zak phase reconstruction for general RM models. Our method of measuring Zak phase in 1D systems can be generalized to identify topological invariants in higher dimensions^{30–34}.



Results

Experimental setup and model

We perform the experiment with the hyperfine levels of the ^{87}Rb D1 line in a standing-wave-coupled EIT configuration, as shown in Fig. 1d (see the complete setup in Supplementary Note 1). A weak probe field propagating in x direction couples the ground state $|g\rangle \equiv |5^2S_{1/2}, F=1\rangle$ to the excited state $|b\rangle \equiv |5^2P_{1/2}, F=2\rangle$. The excited state is also coupled to a metastable state $|a\rangle \equiv |5^2S_{1/2}, F=2\rangle$ by two strong counter-propagating light fields, forming a 1D bipartite superradiance lattice^{23–25}. The absorption spectra of the probe field are used to obtain the Zak phases of the superradiance lattices.

The total Hamiltonian of the superradiance lattice is $H = H_s + H_p + H_f$, where H_s and H_p are interaction Hamiltonians involving the coupling and probe fields, and H_f is the linear potential induced by atomic motion (see Fig. 1e). H_s is the tight-binding Hamiltonian of the RM superradiance lattices^{23–25} (we set $\hbar = 1$ and see Materials and Methods):

$$H_s = \sum_j \left(\Delta_c \hat{a}_{2j}^\dagger \hat{a}_{2j} + \left[\hat{a}_{2j}^\dagger (\Omega_1 \hat{b}_{2j+1} + \Omega_2 \hat{b}_{2j-1}) + \text{H.c.} \right] \right) \quad (2)$$

where Ω_1 and Ω_2 are the Rabi frequencies of the co-propagating and counter-propagating coupling fields, and $\Delta_c = \nu_c - \omega_{ba}$ with ν_c being the coupling field frequency and ω_{ba} being the transition frequency between states $|a\rangle$ and $|b\rangle$. Here $\hat{a}_j^\dagger = \sqrt{1/N} \sum_m |d_m\rangle \langle g_m| \exp(ik_j x_m)$ ($d = a, b$) is the creation operator of the timed-Dicke state²⁶ $|\tilde{d}_j\rangle \equiv \hat{a}_j^\dagger |g_1, g_2, \dots, g_N\rangle$ with wave vector $k_j = k_p + (j-1)k_c \approx jk_c$ (j is an integer) and k_p (k_c) being the probe (coupling) field wave vector amplitude, the index m labels the m th atom at the position x_m , N is the total number of atoms within the velocity range $[\nu - \Gamma/2k_c, \nu + \Gamma/2k_c]$ where ν satisfies the Maxwell distribution and Γ is the decay rate of the state $|b\rangle$ (we neglect the decay of the hyperfine ground state $|a\rangle$). The timed Dicke state $|\tilde{b}_1\rangle$ in the superradiance lattice can be created from the ground state by $H_p = \sqrt{N}\Omega_p e^{-i\Delta_p t} \hat{b}_1^\dagger + \text{H.c.}$, where the probe detuning $\Delta_p = \nu_p - \omega_{bg}$ with ν_p being the probe field frequency and ω_{bg} being the transition frequency between states $|b\rangle$ and $|g\rangle$.

In order to clarify the effect of atomic motion, we show the contribution from atoms with different velocities in x direction. For atoms in each velocity group, the opposite Doppler shifts of the two coupling fields lead to a linear potential (see Fig. 1e) in momentum space^{23,35}:

$$H_f = \delta \sum_j \left[2j\hat{a}_{2j}^\dagger \hat{a}_{2j} + (2j+1)\hat{b}_{2j+1}^\dagger \hat{b}_{2j+1} \right] \quad (3)$$

where the Doppler shift $\delta \approx k_c \nu \approx k_p \nu$ with ν being the velocity of the atoms in x direction.

Wannier-Stark ladders

The energy spectrum of the Hamiltonian $H_s + H_f$ is closely related to the WCs, which are the expected positions of the Wannier functions²² in unit cells. The WCs in the n th unit cell $r_\pm^{[n]}$ for the upper (+) and lower (−) energy bands of H_s are related to the geometric Zak phases by (in unit of distance between neighboring lattice sites, see Fig. 1b)^{20,21}:

$$r_\pm^{[n]} = 2n + \theta_\pm / \pi \quad (4)$$

where the Zak phases $\theta_\pm \equiv i \int_0^{\pi/k_c} dx \langle u_\pm(x) | \partial_x | u_\pm(x) \rangle$ with $|u_\pm(x)\rangle$ being the periodic Bloch functions of H_s in real space and the integration is over the whole Brillouin zone. Therefore, the Zak phases are the fractional parts of the corresponding WCs^{20,21}.

The extended Bloch energy spectra split into discrete WSLs^{15,16,35} with energy spacing proportional to the static force δ when H_f is perturbative (see Fig. 1c):

$$E_\pm^{[n]}(\delta) = \epsilon_\pm + r_\pm^{[n]} \delta \quad (5)$$

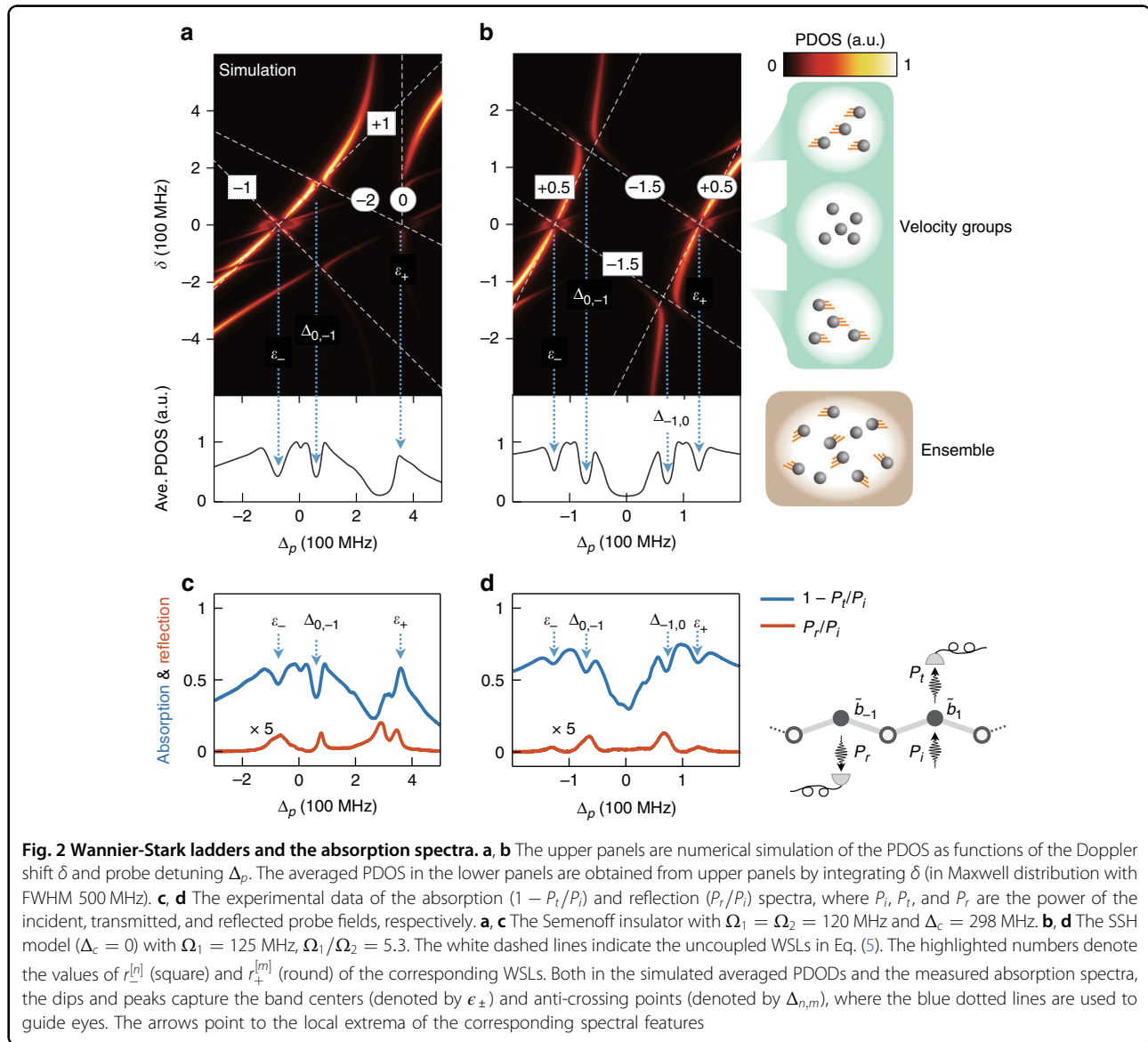
where $\epsilon_\pm$ denote the energies of the Bloch band centers (bc), defined as the average band energies of H_s (See Supplementary Note 2 for the derivation of Eq. (5) and discussion on its validity). From Eqs. (4) and (5), the Zak phases are obtained by $\theta_\pm = \left(\partial E_\pm^{[n]} / \partial \delta - 2n \right) \pi$.

The relation in Eq. (5) can be seen in the upper panels of Fig. 2a, b as functions of δ for two different RM lattices, namely, the Semenoff insulator with $\theta_- = \pi$, $\theta_+ = 0$ (Fig. 2a), and the topological SSH model with half-integer Zak phases $\theta_\pm = 0.5\pi$ (Fig. 2b). Since the Zak phase is gauge-dependent⁷, its value depends on the choice of the unit cell in conventional lattices⁸. However, in our experiments the Zak phase is an observable with a fixed gauge set by the zero-energy site of H_f , which is determined by the Doppler shifts of atoms. This is a significant difference between SLs and conventional lattices³ (see Supplementary Note 3).

The energy spectra of the SLs are shown by the projected density of states (PDOS) on the state $|\tilde{b}_1\rangle$, i.e., $\sum_l \delta_D(\Delta_p - E_l) |\langle \psi_l | \tilde{b}_1 \rangle|^2$, where $\delta_D(\Delta_p - E_l)$ is the Dirac delta function, $|\psi_l\rangle$ and E_l are the eigenstates and eigenenergies satisfying $(H_s + H_f)|\psi_l\rangle = E_l|\psi_l\rangle$ (see Materials and Methods). In the weak force regime where the coupling between the WSLs from different bands are negligible, the spectra follow the linear dependence in Eq. (5) as indicated by the dashed lines. The brightest ladders in the absorption spectra are the ones corresponding to $r_\pm^{[0]} = \theta_\pm / \pi$ in the 0th unit cell, which contains the state $|\tilde{b}_1\rangle$.

Anti-crossing of Wannier-Stark ladders

When a pair of WSLs from different bands ($E_\pm^{[n]}$ and $E_\pm^{[m]}$) have the same energy for a δ , the interband



coupling removes their degeneracy and results in an anti-crossing³⁶ denoted by $ac_{n,m}$. Their positions in the energy-force diagram can be estimated by the degeneracy points of the uncoupled WSLs³⁷ satisfying $E_{-}^{[n]}(\delta) = E_{+}^{[m]}(\delta) = \Delta_{n,m}$, where $\Delta_{n,m}$ is the probe detuning of the corresponding anti-crossing point. The values of $\Delta_{n,m}$ obtained from the experimental absorption spectra are the key to extract the Zak phases.

The optical responses (reflection and absorption) of the superradiance lattice are contributed by all atoms in Maxwell velocity distribution³⁸. We obtain the averaged PDOS in the lower panels of Fig. 2a, b from the corresponding WSLs in the upper panels by integrating δ , which has a Doppler width about 500 MHz and covers all relevant values for the Zak phase reconstruction. We need to emphasize here that our scheme only requires that the

velocity distribution shall be large enough to cover all the relevant anti-crossings. The Maxwell distribution of room-temperature atoms satisfies such a requirement (see the experimental spectra at different temperatures in Supplementary Note 4). The method is equally valid for other velocity distributions, as well as for cold atoms whose velocities can be precisely controlled.

Since the absorption coefficient is proportional to the PDOS²³, the anti-crossings and band centers modify the PDOS drastically and their signatures can be identified in the absorption spectra. The values of $\Delta_{n,m}$ and $\epsilon_{\pm}$ are experimentally measured with the corresponding extrema in the absorption spectra (see exemplary data sets in Supplementary Note 5). As shown in Fig. 2c (and more examples in Fig. 4), the spectra of the two-band SLs are generally featured with four dips, of which two are

associated with the band centers and the rest two are due to anti-crossings. The band centers are generally characterized by dips in the far left and far right wings of the spectra. Only in a special case with zero Zak phase, a band center is featured by a peak in Fig. 2a (see Supplementary Note 6). Between the two band centers, the anti-crossings of WSLs lead to dips in the spectra, reflecting the energy gaps of the anti-crossings. Since we measure the PDOS of the state $|\tilde{b}_1\rangle$ in the 0th unit cell, the major anti-crossings are associated to the Wannier functions localized in the 0th and the neighboring -1 st unit cells, i.e., $\Delta_{-1,0}$ and $\Delta_{0,-1}$.

We also show the reflection spectra in Fig. 2c, d, which is the directional emission from the state $|\tilde{b}_1\rangle$ along $-x$ direction. The reflection spectra also have features characterizing the anti-crossing of the WSLs (e.g., the peaks of reflection spectra correspond to band centers and anti-crossings). On the other hand, they can also be used to study the lattice transport between sites $|\tilde{b}_1\rangle \rightarrow |\tilde{b}_{-1}\rangle$ for lattices in different topological phases, which is out of the scope of the current paper.

Zak phase measurement

In order to measure the Zak phases, we shall quantify the common features in the absorption spectra of lattices with the same Zak phase. In Fig. 3a, we maintain $\Omega_1 = \Omega_2 = 120$ MHz and decrease Δ_c from bottom to top. The Zak phase is the same but the coupling between WSLs increases to widen the anti-crossing gap. We locate the anti-crossing points with its normalized energy:

$$R_{n,m} = (\Delta_{n,m} - \epsilon_-) / (\epsilon_+ - \epsilon_-) \quad (6)$$

On the other hand, according to the geometry of WSLs in the $\Delta_p - \delta$ diagram, $R_{n,m}$ is approximately the normalized ratio of WCs between the WSLs:

$$R_{n,m} \approx \frac{r_-^{[n]}}{r_-^{[n]} - r_+^{[m]}} = \frac{2n\pi + \theta_-}{(2n\pi + \theta_-) - (2m\pi + \theta_+)} \quad (7)$$

In Fig. 3c, we obtain $R_{0,-2} \approx 1/5$, $R_{0,-1} \approx 1/3$. We solve two equations of $\theta_{\pm}$ from the values of $R_{0,-1}$ and $R_{0,-2}$ and conclude that $\theta_- \approx \pi$ and $\theta_+ \approx 0$, as shown in Fig. 3e.

For the SSH models, we keep $\Delta_c = 0$ and tune the Rabi frequencies of the two coupling fields from almost dimerization to the topological phase transition point. The Zak phases are maintained the same while the anti-crossing energy gaps increase from top to bottom in Fig. 3b. The measured $R_{n,m}$ in Fig. 3d agree well with their expected values and the reconstructed Zak phases are obtained, $\theta_{\pm} \approx 0.5\pi$, as shown in Fig. 3f.

For a general RM Hamiltonian, i.e., H_s with $\Omega_1 \neq \Omega_2$ and $\Delta_c \neq 0$, the Zak phases are neither integers nor half-integers. Along the yellow line in the phase diagram in Fig. 4a, we measure the absorption spectra to obtain $R_{0,-1}$

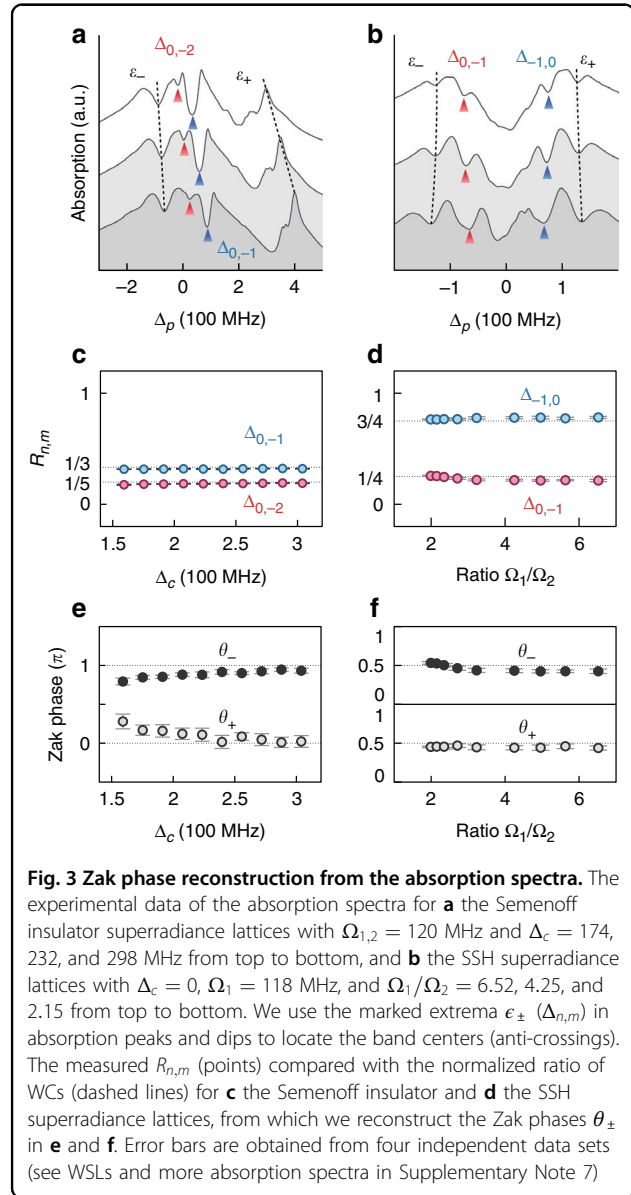
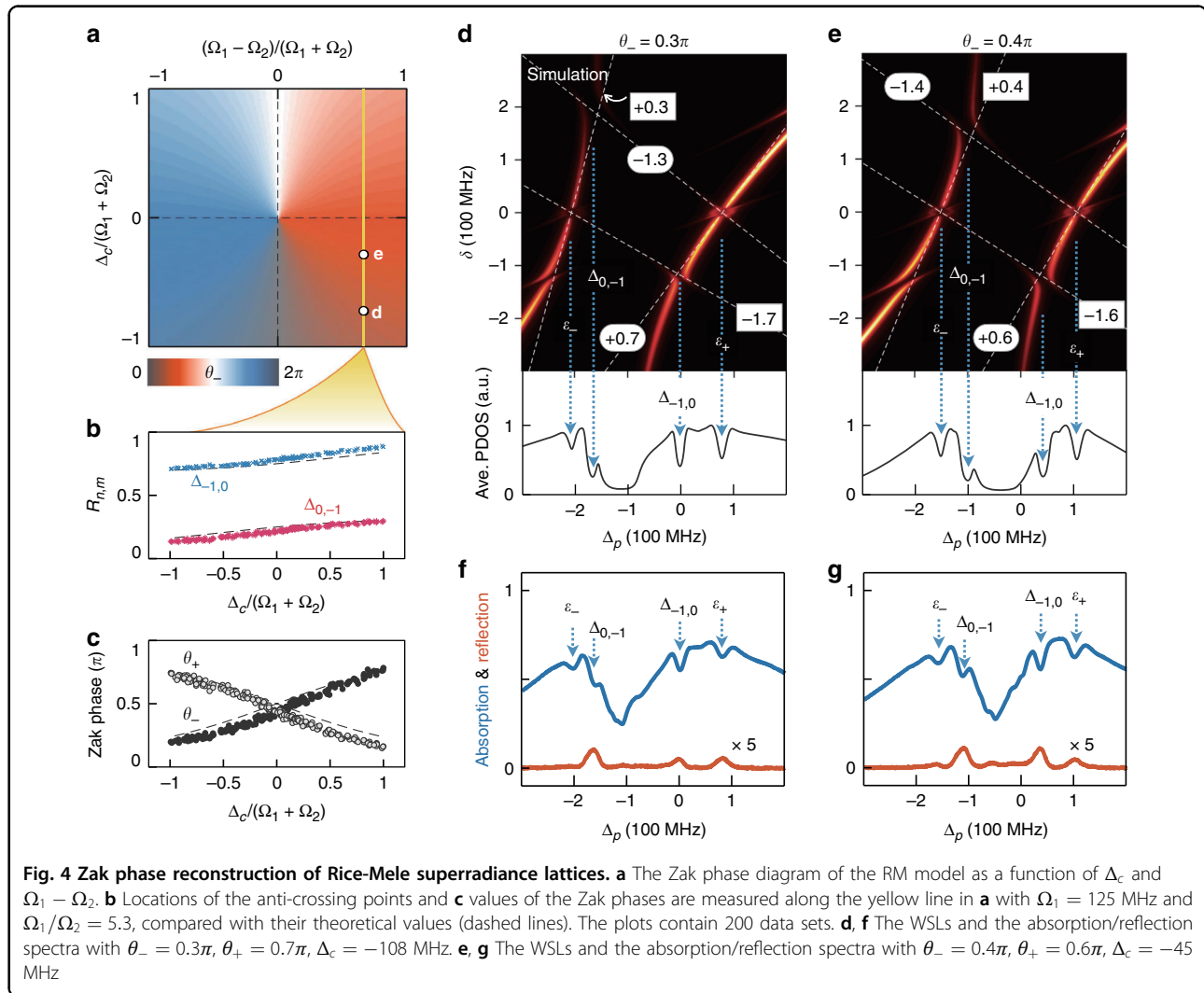


Fig. 3 Zak phase reconstruction from the absorption spectra. The experimental data of the absorption spectra for **a** the Semenoff insulator superradiance lattices with $\Omega_{1,2} = 120$ MHz and $\Delta_c = 174$, 232, and 298 MHz from top to bottom, and **b** the SSH superradiance lattices with $\Delta_c = 0$, $\Omega_1 = 118$ MHz, and $\Omega_1/\Omega_2 = 6.52$, 4.25, and 2.15 from top to bottom. We use the marked extrema $\epsilon_{\pm}$ ($\Delta_{n,m}$) in absorption peaks and dips to locate the band centers (anti-crossings). The measured $R_{n,m}$ (points) compared with the normalized ratio of WCs (dashed lines) for **c** the Semenoff insulator and **d** the SSH superradiance lattices, from which we reconstruct the Zak phases $\theta_{\pm}$ in **e** and **f**. Error bars are obtained from four independent data sets (see WSLs and more absorption spectra in Supplementary Note 7)

and $R_{-1,0}$ for each coupling field detuning Δ_c , as shown in Fig. 4b, and accordingly reconstruct the Zak phases in Fig. 4c, in comparison with the theoretical prediction as indicated by the dashed lines. As an example, we show the WSLs with $\theta_- = 0.3\pi$, $\theta_+ = 0.7\pi$ in Fig. 4d and with $\theta_- = 0.4\pi$, $\theta_+ = 0.6\pi$ in Fig. 4e. The corresponding absorption and reflection spectra are plotted in Fig. 4f, g, respectively.

Discussions

We realize the spectroscopic reconstruction of the Zak phases of momentum-space superradiance lattices. Without trapping atoms or controlling their velocities^{8–10}, we take advantage of the atomic thermal motion^{39,40} to extract geometric phases from the anti-crossings of the WSLs.



Therefore, our result pushes forward the room-temperature quantum simulation of topological matter. Meanwhile, it also paves a way for application of topological physics in optical devices that operate at ambient temperature.

The deviation between the reconstructed Zak phases and the theoretical prediction can be attributed to the following two reasons. First, when the band gap is small, the strong coupling between WSLs leads to a wide anti-crossing energy gap, such that the energy dip does not accurately reflect the location of the anti-crossing (see $\Delta_c \rightarrow 0$ for the Semenoff insulator in Fig. 3e). Second, the competition between H_f and H_s induces a systematic error even for small couplings between WSLs. For the SSH model in Fig. 3f, the force required for the two major anti-crossings is $\delta \approx \pm(\epsilon_+ - \epsilon_-)/2$. The induced potential energy between neighboring sites is comparable to the hopping strengths between lattice sites, such that H_f cannot be treated as a perturbation and the slope of WSLs $\partial E_{\pm}^{[n]}/\partial \delta$ deviates from $r_{\pm}^{[n]}$ (see Supplementary Note 3). The consequence is that

the energy of the anti-crossing $\Delta_{0,-1}$ ($\Delta_{-1,0}$) is always lower (higher) than the degeneracy point predicted by the linear approximation in Eq. (5) (see the difference between spectra extrema $\Delta_{n,m}$ and crossing points of blue dotted lines in Figs. 2a, b and 4d, e), leading to systematic errors in determining the Zak phases.

To improve the accuracy in extracting the spectroscopic features of Zak phases, we are developing a spectral hole burning technique to map out the two-dimensional velocity-dependent absorption spectra of WSLs, as shown in upper panels of Fig. 2a. By using a narrow linewidth saturation field that couples the ground state to an ancillary state, we can selectively bleach the ground state population of atoms with a certain velocity. By comparing the bleached and unbleached absorption spectra, the contribution from atoms with that velocity is obtained.

Our scheme can be generalized to measure multipole moments of higher-order topological insulators^{33,34} by detecting the slopes of WSLs¹⁸, and to measure Chern



numbers by counting the Zak phase winding. In the current framework, two or higher dimensional superradiance lattices¹⁶ can be synthesized by introducing more coupling fields^{41–44}, following similar methods on dimension extension in photonic lattices⁴⁵ and synthetic dimensions^{46–48}. We can use three coupling fields to form a 2D interference pattern in x - y plane (Fig. 5a), illustrating the BZ of a momentum-space honeycomb superradiance lattices⁴¹. We can identify the Chern number $C_{\pm}$ of the upper (+) and lower (−) bands from the winding number of the 1D Zak phase along the perpendicular dimension^{16,24–32}:

$$C_{\pm} = \frac{1}{2\pi} \int_0^L dy \frac{\partial \theta_{\pm}(y)}{\partial y} \quad (8)$$

where $\theta_{\pm}(y)$ is the Zak phase along the x -axis cut of the 2D BZ with a fixed y and L is the length of real-space BZ along the y -axis. In Fig. 5b, we schematically show that $\theta_{\pm}(y)$ can be measured from the absorption spectra of a probe field with a beam size much smaller than L . In order to suppress the paraxial diffraction, we need to ensure L is much larger than the wavelength by minimizing the angle between the two copropagating coupling fields. After collecting the $\theta_{\pm}(y)$, we determine the Chern number of the 2D superradiance lattices by counting how many times it winds within L , as shown in Fig. 5c.

With the ability of measuring geometric phases in SLs, a promising direction in the next stage is to introduce interactions between atoms, e.g., by using Rydberg states⁴⁹. It is particularly interesting to notice that the short-range interaction in real space has long-range effect in momentum-space SLs, which is difficult to realize in real-space lattices⁵⁰.

In conclusion, we develop a method of reconstructing Zak phases from the anti-crossings of WSLs by measuring the Doppler-broadened absorption spectra of room-temperature superradiance lattices. This method can be directly generalized to measure Chern numbers by counting the Zak phase windings in 2D lattices¹⁶ with more coupling fields^{41–44}. Our method can also be implemented in cold atoms²⁴ by controlling the atomic velocity to track the peak shifting, as sketched in Fig. 1c. We can also use a hole burning technique to obtain the WSL of atoms with different velocities. Our results pave the way to detect multipole moments in higher-order topological insulators^{18,33,34}.

Materials and methods

Effective Hamiltonian

Here we derive the effective Hamiltonian $H = H_s + H_f + H_p$. It is convenient to work with the master equation in the inertial reference frame of the moving atoms.

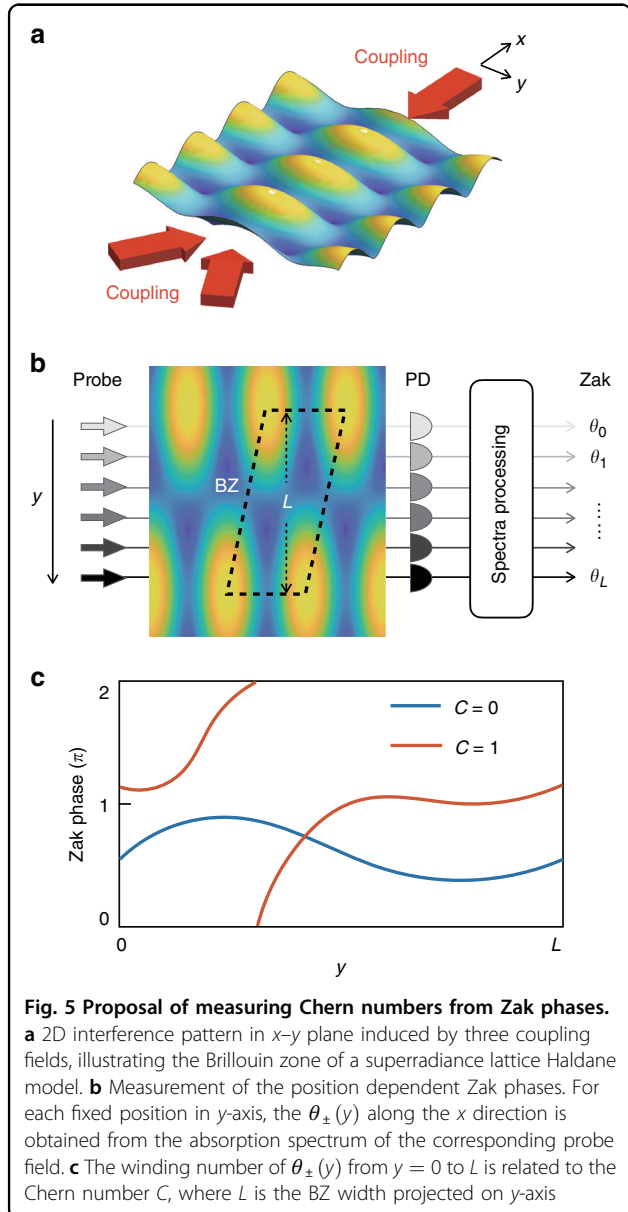


Fig. 5 Proposal of measuring Chern numbers from Zak phases.

a 2D interference pattern in x - y plane induced by three coupling fields, illustrating the Brillouin zone of a superradiance lattice Haldane model. **b** Measurement of the position dependent Zak phases. For each fixed position in y -axis, the $\theta_{\pm}(y)$ along the x direction is obtained from the absorption spectrum of the corresponding probe field. **c** The winding number of $\theta_{\pm}(y)$ from $y = 0$ to L is related to the Chern number C , where L is the BZ width projected on y -axis

The Rabi frequencies of the two plane wave components of the coupling field are Ω_1 (x directional) and Ω_2 ($-x$ directional), and the corresponding Doppler shifted frequencies are $\nu_c - \delta$ and $\nu_c + \delta$, respectively. For the probe field propagating along x direction, the Rabi frequency and Doppler shifted frequency are Ω_p and $\nu_p - \delta$, respectively. The original Hamiltonian of an ensemble of Λ -type three-level atoms in EIT is written as:

$$H' = \omega_{bg}|b\rangle\langle b| + \omega_{ag}|a\rangle\langle a| + (\Omega_1 e^{-i(\nu_c - \delta)t + ik_c x}|b\rangle\langle a| + \Omega_2 e^{-i(\nu_c + \delta)t - ik_c x}|b\rangle\langle a| + \text{H.c.}) + (\Omega_p e^{-i(\nu_p - \delta)t + ik_p x}|b\rangle\langle g| + \text{H.c.}) \quad (9)$$



In order to eliminate the dynamic phase factors, we transform the Hamiltonian into the interaction picture:

$$V = U^{-1}H'U - S = \Delta_c|a\rangle\langle a| + (\Omega_p e^{-i(\Delta_p - \delta)t + ik_p x}|b\rangle\langle g| + \text{H.c.}) + (\Omega_1 e^{i\delta t + ik_c x}|b\rangle\langle a| + \Omega_2 e^{-i\delta t - ik_c x}|b\rangle\langle a| + \text{H.c.}) \quad (10)$$

where $U = \exp(-iSt)$ and $S = \omega_{bg}|b\rangle\langle b| + (\omega_{bg} - \nu_c)|a\rangle\langle a|$.

In the experiments, the powers of the coupling fields are much larger than that of the probe field, and the probe field is far below the saturation strength, i.e., $\Omega_{1,2}, \Gamma \gg \Omega_p$. Therefore, we only keep the first order of Ω_p but keep all orders of $\Omega_{1,2}$. The relevant dynamical equations that govern the evolution of coherence ρ_{ag} and ρ_{bg} are:

$$\begin{aligned} \dot{\rho}_{ag} &= -i[(\Omega_1 e^{-i\delta t - ik_c x} + \Omega_2 e^{i\delta t + ik_c x})\rho_{bg} + \Delta_c \rho_{ag}] - \gamma_a \rho_{ag}, \\ \dot{\rho}_{bg} &= -i[(\Omega_1 e^{i\delta t + ik_c x} + \Omega_2 e^{-i\delta t - ik_c x})\rho_{ag} + \Omega_p e^{-i(\Delta_p - \delta)t + ik_p x}] - \gamma_{bg} \rho_{bg} \end{aligned} \quad (11)$$

where $\gamma_{bg} = \Gamma/2 + \gamma_b$ is the decoherence rate of ρ_{bg} and γ_i is the dephasing rate of the level $|i\rangle$. The general solutions are assumed as:

$$\begin{aligned} \rho_{ag} &= \sum_j \rho_{ag}^{[2j]} e^{ik_{2j}x'}, \\ \rho_{bg} &= \sum_j \rho_{bg}^{[2j+1]} e^{ik_{2j+1}x'} \end{aligned} \quad (12)$$

where $k_j = k_p + (j-1)k_c \approx jk_c$, $x' = x + \delta/k_c t = x + \nu t$ being the position of atoms in motion. In the weak excitation limit $\rho_{ag}^{[2j]}, \rho_{bg}^{[2j+1]} \ll 1$, the wavefunction of a single atom at x_m is approximately $|\Psi_m\rangle \approx \rho_{ag}|a_m\rangle + \rho_{bg}|b_m\rangle + |g_m\rangle$. Therefore, the wavefunction of the whole atomic ensemble reads:

$$\begin{aligned} |\Psi\rangle &= \prod_m |\Psi_m\rangle \\ &\approx \sum_m \sum_j \rho_{ag}^{[2j]} e^{ik_{2j}x'_m} |g_1 g_2 \dots a_m \dots g_N\rangle \\ &\quad + \sum_m \sum_j \rho_{bg}^{[2j+1]} e^{ik_{2j+1}x'_m} |g_1 g_2 \dots b_m \dots g_N\rangle + |G\rangle \\ &= \sum_j (\alpha_{2j} \hat{a}_{2j}^\dagger + \beta_{2j+1} \hat{b}_{2j+1}^\dagger + 1) |G\rangle \end{aligned} \quad (13)$$

where we use the definition of the collective ground state $|G\rangle = |g_1 g_2 \dots g_N\rangle$ and the probability amplitudes of the timed Dicke states $\alpha_{2j} = \sqrt{N} \rho_{ag}^{[2j]}$, $\beta_{2j+1} = \sqrt{N} \rho_{bg}^{[2j+1]}$. Combining Eqs. (11)–(13), we write the dynamic equation formally:

$$i \frac{d}{dt} |\Psi\rangle = (H - i\hat{\gamma}) |\Psi\rangle \quad (14)$$

with the effective Hamiltonian:

$$H = \sum_j \left((\Delta_c + 2j\delta) \hat{a}_{2j}^\dagger \hat{a}_{2j} + \delta(2j+1) \hat{b}_{2j+1}^\dagger \hat{b}_{2j+1} \right) + \left[\sum_j \left(\Omega_1 \hat{a}_{2j}^\dagger \hat{b}_{2j+1} + \Omega_2 \hat{a}_{2j} \hat{b}_{2j+1}^\dagger \right) + \sqrt{N} \Omega_p e^{-i\Delta_p t} \hat{b}_1^\dagger + \text{H.c.} \right]$$

and the dissipation operator:

$$\hat{\gamma} = \sum_j \left(\gamma_a \hat{a}_{2j}^\dagger \hat{a}_{2j} + \gamma_{bg} \hat{b}_{2j+1}^\dagger \hat{b}_{2j+1} \right)$$

Absorption and PDOS

The induced polarization of the state $|\Psi\rangle$ in Eq. (13) is defined as:

$$P = \langle \Psi | e r | \Psi \rangle = \sum_j \sum_m \mu \rho_{bg}^{[2j+1]} e^{ik_{2j+1}x'_m} + c.c. \quad (15)$$

where $\mu = \langle g | e r | b \rangle$ is the single atom dipole moment. Since the atoms are homogeneously distributed, the polarization density as a function of position reads:

$$P(x) = n \sum_j \mu \rho_{bg}^{[2j+1]} e^{ik_{2j+1}x'} + c.c. \quad (16)$$

where n is the atomic density. Therefore, the susceptibility of the atoms is:

$$\chi = \frac{P(x)}{\epsilon_0 E_p e^{ik_p x'}} = \sum_j \chi^{[j]} e^{ik_{2j}x'} \quad (17)$$

The optical absorption coefficient A is related to the 0th-order component (linear) of the susceptibility in Eq. (17), and further connected to PDOS of the superradiance lattices:

$$\begin{aligned} A &\propto \text{Im} \chi^{[0]} \propto \text{Im} \beta_1 \\ &\propto \text{Im} \left\langle \tilde{b}_1 \left| \frac{1}{(\Delta_p + i\gamma_{bg}) - H_f(\nu) - H_s} \right| \tilde{b}_1 \right\rangle \\ &= \text{Im} \sum_l \frac{|\langle \psi_l(\nu) | \tilde{b}_1 \rangle|^2}{(\Delta_p + i\gamma_{bg}) - E_l(\nu)} \\ &\approx -\pi \sum_l \delta_D(\Delta_p - E_l(\nu)) |\langle \psi_l(\nu) | \tilde{b}_1 \rangle|^2 \end{aligned} \quad (18)$$

where we take advantage of the Green's function approach^{51–53} and the last step is valid when the decoherence rate tends to zero compared with the band width. Here H_f , $|\psi_l\rangle$, and E_l are functions of ν . The power of the transmitted probe field is $P_t = P_i e^{-As}$, where s is the length of the Rb vapor cell.

Acknowledgements

The authors thank Luqi Yuan for useful discussions. This work was supported by the National Natural Science Foundation of China (Grants Nos. 11874322, 11934011 and U21A20437), the National Key Research and Development



Program of China (Grants Nos. 2019YFA0308100, 2018YFA0307200 and 2017YFA0304202), Zhejiang Province Key Research and Development Program (Grant No. 2020C01019), the Strategic Priority Research Program of Chinese Academy of Sciences (Grant No. XDB28000000), the Fundamental Research Funds for the Central Universities and Innovation Program for Quantum Science and Technology (Grant No. 2021ZD0303200). We gratefully acknowledge HZWTECH for providing computation facilities.

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H.C. and D.W.W. conceived the project. H.C. and X.X. designed the experiment. J.W., X.X., C.X., and G.Q. built the experimental setup and carried out the measurement. R.M. and H.C. did numerical simulation. R.M., J.W., X.X. and H.C. performed data analysis. D.W.W. and S.Y.Z. supervised the research. H.C. and D.W.W. wrote the manuscript with comments and contributions from all authors.

Conflict of interest

The authors declare no competing interests.

Supplementary information The online version contains supplementary material available at <https://doi.org/10.1038/s41377-022-00990-7>.

Received: 23 May 2022 Revised: 20 September 2022 Accepted: 21

September 2022

Published online: 09 October 2022

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